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7.3 (a) $\left(\frac{P}{Ka}\right)\sin Ka + \cos Ka = \cos ka$ (Kronig-Penny soln.)

with $k=0$, we have $\frac{P}{Ka}\sin Ka + \cos Ka = 1$,

$$P \sin Ka = Ka[1-\cos Ka]$$

$$\cos Ka = \cos 2\frac{Ka}{2} = \cos^2\frac{Ka}{2} - \sin^2\frac{Ka}{2}$$

$$\Rightarrow \quad 1-\cos Ka = \cos^2\frac{Ka}{2} + \sin^2\frac{Ka}{2} - \cos^2\frac{Ka}{2} + \sin^2\frac{Ka}{2}$$

$$= 2\sin^2\frac{Ka}{2}$$

$$\Rightarrow \quad P\sin Ka = Ka\, 2\sin^2\frac{Ka}{2}$$

$$2P\sin\frac{Ka}{2}\cos\frac{Ka}{2} = Ka\, 2\sin^2\frac{Ka}{2}$$

$$\cancel{2}P\cos\frac{Ka}{2} = Ka\,\cancel{2}\sin\frac{Ka}{2}$$

$$\Rightarrow \boxed{\frac{P}{Ka} = \tan\frac{Ka}{2}}$$

Use approximation for $\frac{Ka}{2}$ small, $\tan\frac{Ka}{2} \simeq \frac{Ka}{2}$,

$$\frac{P}{Ka} \simeq \frac{Ka}{2}, \quad K^2 \simeq \frac{2P}{a^2}, \quad K = \sqrt{\frac{2P}{a^2}},$$

$$\varepsilon = \frac{\hbar^2 K^2}{2m} \Rightarrow \varepsilon = \frac{2P\hbar^2}{2ma^2} = \boxed{\frac{P\hbar^2}{ma^2}}$$

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7.3(b). For $k = \frac{\pi}{2}$, we have from the Kronig-Penny soln.

$$\frac{P}{Ka}\sin Ka + \cos Ka = -1$$

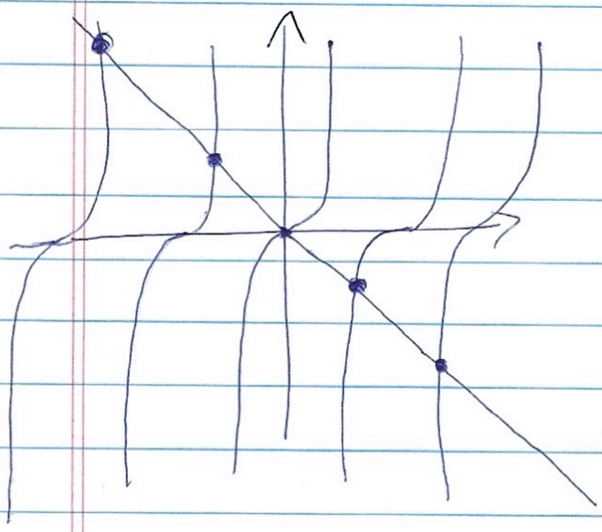
$$P\sin Ka = -Ka\left[1+\cos Ka\right].$$

$$1+\cos Ka = 1+\cos 2\frac{Ka}{2} = 1+\cos^2\frac{Ka}{2} - \sin^2\frac{Ka}{2}$$
$$= 2\cos^2\frac{Ka}{2}.$$

$$\Rightarrow P\sin Ka = -Ka\, 2\cos^2\frac{Ka}{2}.$$
$$2P\cos\frac{Ka}{2}\sin\frac{Ka}{2} = -2Ka\cos^2\frac{Ka}{2}.$$
$$P\sin\frac{Ka}{2} = -Ka\cos\frac{Ka}{2}.$$
$$\boxed{P\tan\frac{Ka}{2} = -Ka}$$

This equation gives allowed values of K. For simplicity we now consider solutions for $P\tan x = -x$, graphing:

$y = P\tan x$



$y = -x$.

we see that the solutions will tend to have same periodicity as tan. Thus we expect

$$\Delta K \approx \frac{2\pi}{a}.$$

$$\Rightarrow \Delta \varepsilon \approx \frac{\left(\frac{2\pi}{a}\right)^2\hbar^2}{2m}$$
$$\approx \boxed{\frac{2\pi^2\hbar^2}{a^2m^2}}$$

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